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EMD for Technical Analysis (Preprint 2019)

***Abstract* — The decomposing of stock market prices into a set of sub waves with different frequencies by the discrete Fourier transform provides unstable and unsatisfying results. The Empirical Mode Decomposition (EMD) is a decomposing algorithm which uses sub waves with amplitude modulation and frequency modulation, leading to better adapted analyze results. As a non causal algorithm the EMD bears challenges for the technical analysis. Nevertheless, the EMD provides valuable information about market data. This paper presents the EMD algorithm and its features within the context of technical analysis.**

***Index Terms*—EMD, technical analysis, spectral analysis, price sequence**

I. Introduction

Stock market prices are time discrete signals which are nonlinear and non stationary. One of the most important models in technical analysis is the wave model of the market [1]. The wave model assumes that the final market price is the sum of several sub oscillating waves and a final trend component, which is not oscillating. The sub waves are also price sequences which oscillate with different period lengths. This model can be exactly calculated by using digital signal processing techniques [1, 2].

The decomposing of the price sequence into a set of sub waves with different frequencies by the discrete Fourier transform, which is the most common spectral analysis transform, e.g. by the Goertzel algorithm [3], provides only unsatisfying results. The main problem is that the discrete Fourier transform uses sine waves with constant frequencies, which are not optimal in modeling the price waves of the market, as the frequencies of the sub waves are not constant but change permanently, leading to harmonic waves which disappear within less than a period in the output of the Fourier transform. Furthermore, the Fourier transform assumes that the input signal is linear and stationary [4], but stock price sequences are neither linear nor stationary. The Empirical mode decomposition (EMD) is an algorithm which operates on the local time scale of the data and can handle nonlinear and non stationary data [5]. Their components consist of oscillating waves with slightly changing frequencies and changing amplitudes, which are more applicative in modeling the price waves. Unfortunately, the EMD is a non causal algorithm, which causes problems when used for technical analysis. Subsequently the algorithm of the EMD is explained, afterwards the properties, advantages and disadvantages of the EMD within the context of technical analysis are unveiled.

II. The EMD Algorithm

The empirical mode decomposition (EMD) is an algorithm, which dissects a price sequence *x(t)* into a number of sub waves plus a final residuum between the start of the price sequence t_s and the end of the price sequence at t_e. The sub waves are called Intrinsic Mode Functions (IMF), which have slightly changing frequencies and changing amplitudes. The final residuum *FR* is non oscillating.

Therefore, the price sequence *x(t)* can be constructed by the sum of the IMF_i plus *FR*:

$$x(t)=\sum_{i=0}^{n-1} IMF_i(t)+FR \tag{1}$$

The EMD algorithm is described in detail in several publications [5, 6, 7], therefore here a brief description from a programmer's perspective is given. The algorithm works top/down, and has an outer and an inner loop. It starts the outer loop with the input signal as the starting signal. From this signal the first IMF, called IMF_0, is calculated. Then the algorithm continues with the difference of the

Manuscript submitted September 25, 2018.
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```
{ Initializing }
Signal := Input_Signal;

{ Calculate until final residuum is reached }
I := 0;
While (Signal is oscillating) do
  Begin { While }
    { Save old signal }
    Temp_Signal := Signal;

    { Sifting loop }
    While (Signal is oscillating) and
   (Number_of_Maxima (Signal) + Number_of_Maxima (Signal) >
    Number_of_Zero_Crossings (Signal) + 1) and
    (Sum (Signal) > Threshold) do
      Begin { While }
        Local_Maxima := Calculate_Local_Maxima (Signal);
        Local_Minima := Calculate_Local_Minima (Signal);

        Maxima_Signal := Calculate_Cubic_Splines (Local_Maxima);
        Minima_Signal := Calculate_Cubic_Splines (Local_Minima);

        Mean := (Maxima_Signal - Minima_Signal) / 2;

        Signal := Signal - Mean;
      End; { While }

    { Save IMF }
    IMF (I) := Signal;

    Signal := Temp_Signal - Signal;
    I := I + 1;
  End; { While }

{ Save FR }
FR := Signal;
```

Figure 1: EMD algorithm

former signal minus IMF_0 as the new signal. This outer loop is continued until the difference is not oscillating any more, id est the number of maxima or the number of minima is smaller than 2.

The inner loop calculates the IMF, which is called sifting process. The sifting process calculates the local maxima and the local minima of the signal. Next, a cubic spline which uses the local maxima as its knots is calculated and used as the upper part of an envelope. Then a cubic spline which uses the local minima as its knots is calculated and used as the lower part of the envelope. The middle of the envelope is defined as the mean signal. The boundary conditions at the beginning of the price sequence t_s and at the end of the price sequence t_e need a special treatment. At the border the cubic spline is not restricted by any more knots so it tends to raise or decrease quite strongly. In order to achieve a better alignment, the last two knots at each border of the spline are mirrored behind the border. This way, the route of the spline shows a

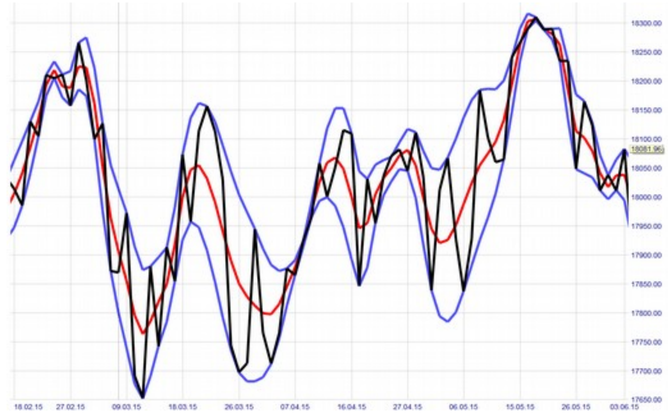

Figure 2: First mean with envelope based on DOW-JONES (signal: black, envelope: blue, mean: red)

Figure 3: IMF_0 based on DOW-JONES

more realistic behavior. Finally the difference between the signal and the mean is used as the new signal and the inner loop is repeated until the number of maxima plus the number of minima is smaller or equal than the number of zero crossings of the signal. This condition ensures that between two zero crossings are only one maximum or one minimum respectively, which is important as the IMF should be an orthogonal signal [5]. The second condition of the inner loop requests that the average mean of the signal is zero, id est the signal has no non zero trend part left, which is also important for an orthogonal signal [5]. *Figure 1* lists the simplified algorithm as a structured program. Note that the variables Input_Signal, Signal, Mean, Local_Maxima etc. are series, id est arrays of numbers and not single numbers. *Figure 2* displays the first mean (red line) together with the envelopes (blue lines) and the original price sequence *x(t)* (black line). *Figure 2* until 7 are all calculated for the same time interval on the x-axis on a daily price

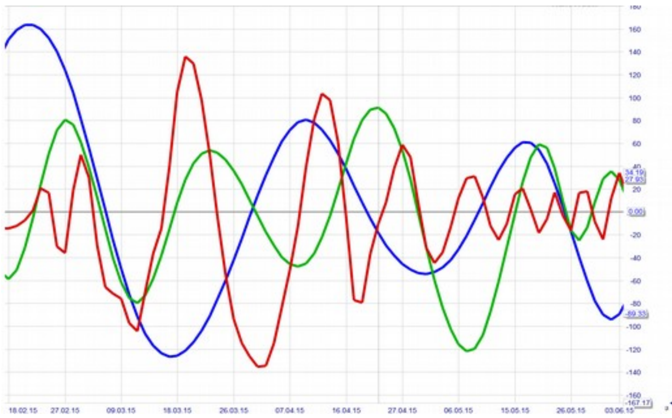

Figure 4: IMF_1 (red), IMF_2 (green) and IMF_3 (blue) based on DOW-JONES between 18.02.2015 and 03.06.2015

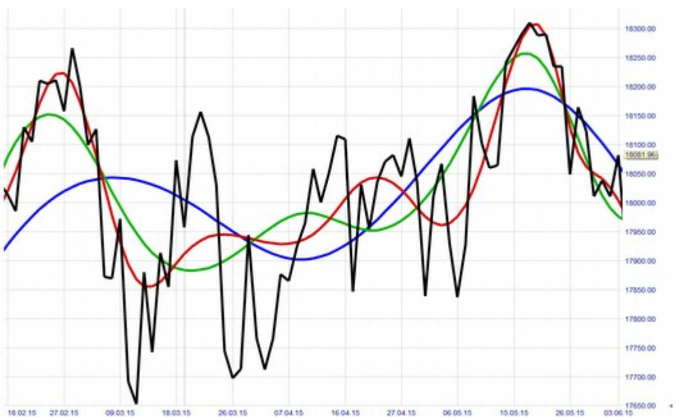

Figure 5: $Mean_1$ (red), $Mean_2$ (green) and $Mean_3$ (blue) based on DOW-JONES between 18.02.2015 and 03.06.2015

frame. In *Figure 3* IMF_0 is drawn as a green line. Note that this IMF has a short period as it is the first IMF. IMF_1 (red), IMF_2 (green) and IMF_3 (blue) are shown in *Figure 4*. The higher IMFs have longer periods and get smoother than lower IMFs. For all IMFs the amplitude is not constant (amplitude modulation) and the period length is changing slightly (frequency modulation). In *Figure 5* $mean_1$ (red), $mean_2$ (green) and $mean_3$ (blue) are displayed. All means render the peak in May 2015 very well, which is an effect of the approximation process, by taking all prices into account, and not only prices before May 2015.

III. Properties of EMD in the Context of Technical Analysis

A. Computational Complexity

Most indicators of the technical analysis [8] have a computational time complexity of $O(n)$; examples are:

- standard (arithmetic) moving average,
- the exponential moving average,
- low, high and band-pass filters,
- MACD (Moving Average Convergence/Diverg.).

Some have a complexity of $O(n*m)$ with m equals the span of an interval for example for the standard deviation like Bollinger Bands [8]. The EMD algorithm has a complexity of $O(n^2*p*q)$ with p equals the number of IMFs (outer loop) and q equals the number of iterations of the sifting process (inner loop). The quadratic factor n^2 is a result of the spline interpolation [9]. This complexity has consequences for the trading, as depending on the number of prices inside the price sequence the calculation of the EMD might need several seconds to be calculated. If the EMD is used for real time trading, where several price signals can occur within a second, the system might hang as the calculation for one new price tick need to calculate all IMFs again and this might take longer than the left over time until the next price tick arrives. If the EMD is used for longer time frames like days or weeks, the time for calculation is usually not a problem.

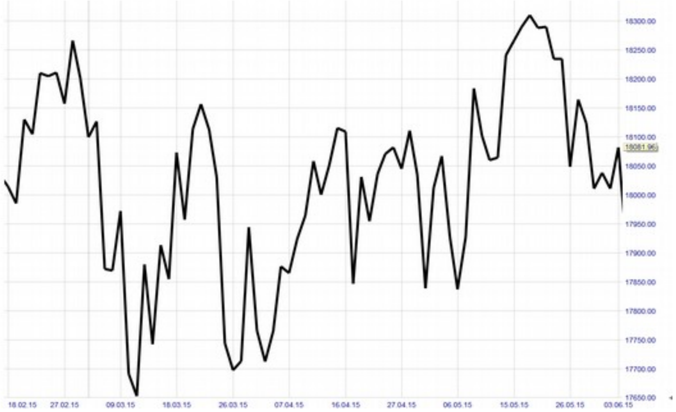

Figure 6 (a): Price sequence DOW-JONES

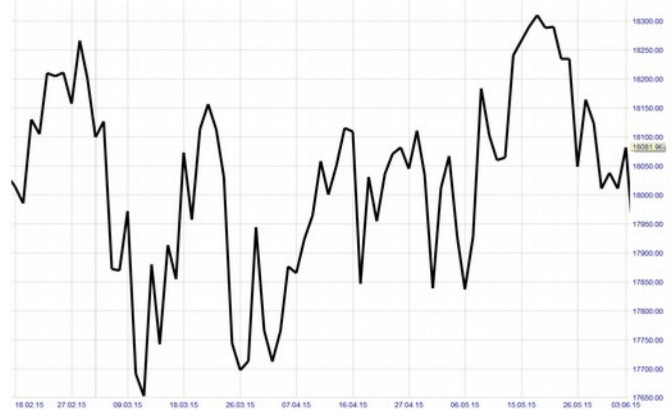

Figure 7 (a): Price sequence DOW-JONES

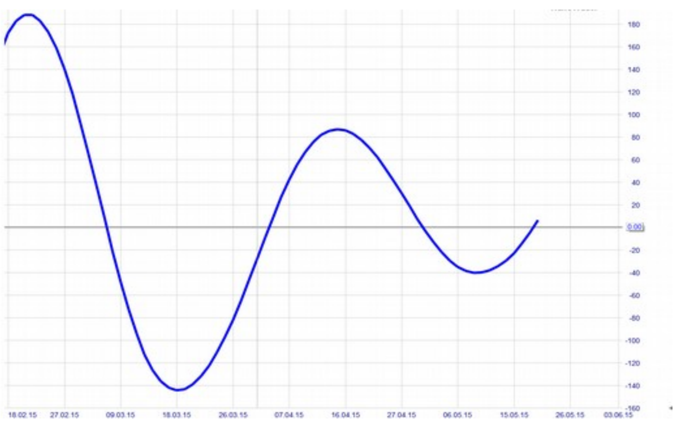

Figure 6 (b): IMF_3 between 18.02.2015 and 20.05.2015

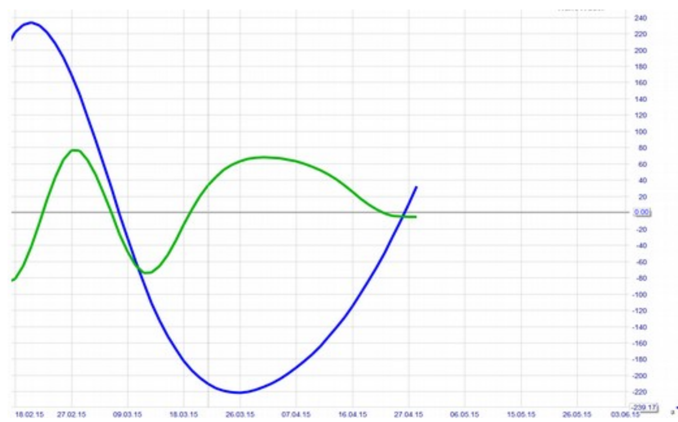

Figure 7 (b): IMF_2 and IMF_3 between 18.02 and 28.04.2015

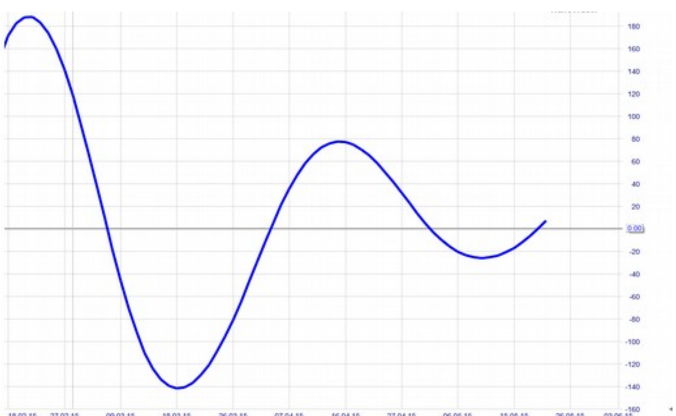

Figure 6 (c): IMF_3 between 18.02.2015 and 21.05.2015

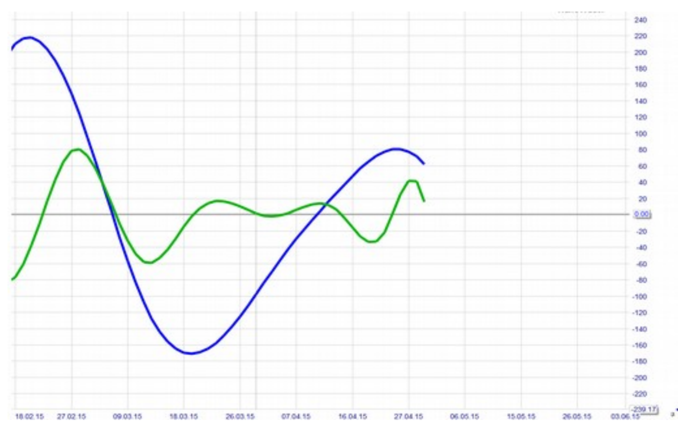

Figure 7 (c): IMF_2 and IMF_3 between 18.02 and 29.04.2015

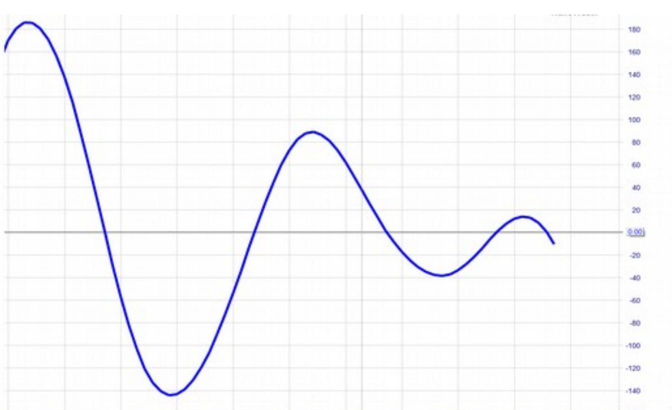

Figure 6 (d): IMF_3 between 18.02.2015 and 22.05.2015

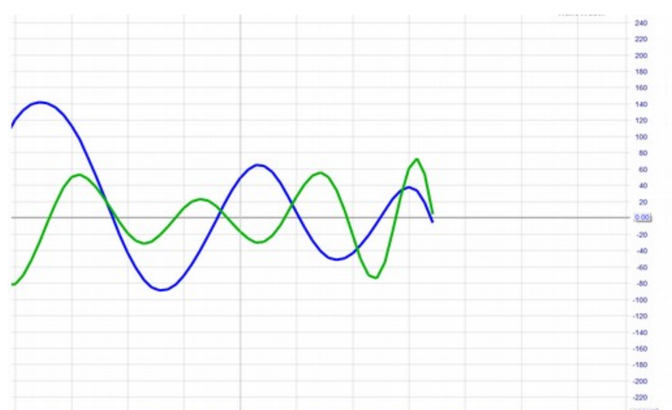

Figure 7 (d): IMF_2 and IMF_3 between 18.02 and 30.04.2015

IMF	DOW JONES DAILY	DAX DAILY	DOW JONES HOURLY	DAX HOURLY
0	3	3	3	3
1	7	7	6	6
2	16	15	11	11
3	33	35	21	21
4	83	89	39	42
5	208	186	74	97
6	-	-	163	215
7	-	-	320	478

Table 1: Average period lengths of the IMFs for the DOW-JONES and DAX from 2014 until 2017 for daily and for hourly time frames

B. Consistency of results

The basis of the EMD is an approximation procedure. Since all prices of the sequence are taking into account for the approximation and since the mirrored prices at the right boundary, which is moving one point to the right with every new price, are replaced by the new prices, the graph of the IMFs can change not only for the points at the right boundary but for the whole price sequence. *Figure 6* presents the change of IMF_3 for three consecutive prices between 20.05.2018 and 22.05.2015. At *Figure 6 (a)* the price sequence is drawn. *Figure 6 (b)* contains IMF_3 on 20.05.2015. The graph of IMF_3 for the next day is shown in *Figure 6 (c)*. There is only a small difference in the shape of IMF_3. One day later, on 21.05.2015 in *Figure 6 (d)*, the graph of IMF_3 changes noticeably on the right border. Note that not only the values of the last few days but about all values for the last two month have changed. *Figure 7 (a) - (d)* reveals an even more drastic example for IMF_2 and IMF_3 between 28.04.2015 and 30.04.2015. Between 28.04.2015 and 29.04.2015 IMF_3 changes its values for about one month and IMF_2 changes its values for about two months. The next day on 30.04.2015 in *Figure 7 (d)* about all values of IMF_2 and IMF_3 have changed for the last four months. But not only the amplitudes have changed but also the number of maxima and minima. On *Figure 7 (c)* for the 29.04.2015, IMF_3 has two maxima and one minima. One day later on *Figure 7 (d)*, IMF_3 has three maxima and two minima, which means that the frequency of IMF_3 has changed. If entry and exit signals are calculated based on the graph of IMF_2 and IMF_3, this signals would have changed vigorously too. Therefore, the consistency of IMFs

Figure 8: Price sequence DOW-JONES for 2017

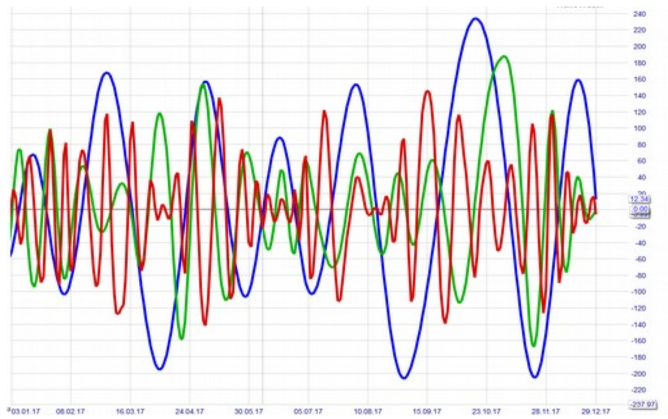

Figure 9: IMF_1 (red), IMF_2 (green) and IMF_3 (blue) based on DOW-JONES in 2017

signals between two consecutive points of time is not ensured, the graph of the IMFs can be much different for two consecutive points of time.

C. Frequencies

Table 1 shows the period length, the reciprocal value of the frequency, for different IMFs and for different price series. In technical analysis the period length is more widely used than the frequency value. The second column displays the period lengths for the Dow Jones Industrial Average (DOW-JONES) for daily time frames between 01.01.2014 and 31.12.2017. The next column contains the period values for the German DAX between 2014 and 2017 in daily time frames. The fourth column implicates the period length for the DOW-JONES between 2014 and 2017 for hourly time frames and the last column includes the results for the hourly time frames of the DAX between 2014 and 2017. Even though the price sequence of the DOW-JONES and the DAX are quite different between 2014 and 2017, the period lengths of the IMFs for the daily time frames as well as the period

IMF	DOW JONES DAILY	DAX DAILY	DOW JONES HOURLY	DAX HOURLY
0	119	114	24	24
1	129	138	28	28
2	238	230	39	37
3	278	339	52	51
4	634	437	72	83
5	755	764	103	124
6	-	-	148	203
7	-	-	225	263

Table 2: Average amplitudes of the IMFs for the DOW-JONES and DAX from 2014 until 2017 for daily and for hourly time frames

length of the hourly time frames are quite similar. The quotient IMF_i/IMF_{i-1} between two consecutive IMFs is between 1.9 and 2.5. For daily time frames the quotients are a little bit larger on average than for the hourly time frames.

D. Number of IMFs

Since the quotient between two consecutive IMFs is on average around 2.3 and since the first period length starts with 3, the number of IMFs for each price sequence depends mainly on the size of the sequence, id est on the number of prices. The daily time frames from 2014 to 2017 contain about 1,000 prices, which makes for 6 IMFs, whereas the hourly time frames from 2014 to 2017 include about 17,000 prices and this causes 8 IMFs. If the interval size would have been expanded or the time frame reduced, both leading to more prices, more IMFs would emerge. On the other side, for a reduced interval or a larger time frame, meaning less prices, the number of IMFs would decline.

E. Amplitudes

The daily DOW-JONES price sequence for 2017 is depicted in *Figure 8*. *Figure 9* portrays the amplitudes of IMF_1 (red), IMF_2 (green) and IMF_3 (blue) for the daily DOW-JONES in 2017. The image is similar to *Figure 4* but for a larger time interval. Again, the amplitudes for each IMF are not constant but change from period to period. On average IMF_3 has a larger amplitude than IMF_2, which has a larger amplitude than IMF_1. *Table 2* reveals the average amplitudes for the DOW-JONES and the DAX between 01.01.2014 and 31.12.2017 for daily and hourly time frames. Similar to the Dow Theory, were the slower waves are the main waves (main movement), which have larger amplitudes than the faster waves (medium and short swing) [10], each IMF_i has a larger amplitude than its predecessor IMF_{i-1} for both markets and for both time frames. Again the similarity of the average amplitude sizes between DOW-JONES and DAX for the daily time frame as well as for the hourly time frame is striking. For the daily DOW-JONES two IMFs are eminent, since the amplitude for this IMF and the amplitude of its smaller predecessor have a greater quotient than on average: IMF_2 and IMF_4. For the daily DAX the major IMFs are IMF_2 and IMF_5. For the hourly DOW-JONES the major IMF changes to IMF_7 and for the hourly DAX the major IMFs are IMF_4 and IMF_6.

IV. Using EMD for trading

A. Disadvantages of EMD for trading

As already mentioned above, the IMF signals can change considerably between two consecutive points of time. The value of IMF_i at time t_0 with $t_s < t_0 < t_e$ not only depends on $x(t)$ for all $t <= t_0$ but also on $x(t)$ with $t > t_0$. Therefore the EMD indicator is not a causal indicator. A non causal indicator causes problems when used for real time trading, as trading signals based on the indicator in the past tends to jump from one place to another place in the past between two consecutive points of time. Therefore a non causal indicator can not be used for back tests as it disturbs the results of the back test. Some analysts used EMD for back-testing and achieved a percentage for winning trades to loosing trades between 90% to 100% [6] by simply opening a long trade when the IMF has a local minimum and closing the trade when the IMF reaches a local maximum. These results should be treated with caution as the final positions of the local maximums and minimum are not fixed inside the sequence of prices but can change with every new price. Furthermore, it is doubtful to use the IMFs for predicting future price development by using forward projections of the IMFs as damped harmonic oscillations [7] as the IMFs values on the end of the price sequence at t_e change and jump much stronger than IMF values in the middle or at the beginning of the sequence between two consecutive points of time. Therefore, the EMD should be used with care for generating trading signals, back tests or predicting future price development.

B. Advantages of EMD for trading

Despite the fact that the EMD has many disadvantages for real time trading mainly because of the missing causality, it offers some properties which are very useful for the technical analysis. The EMD produces a spectral decomposition of the market price sequence into set of sub waves, which is much more appropriate for market prices than spectral decompositions from Fourier transform [4], Goertzel algorithm [3], MESA [11] or Autocorrelation [12]. The main difference of the results between these algorithms and the EMD is that the sub waves of the EMD have an amplitude modulation and a frequency modulation. The sub waves from the EMD start at the beginning of the price sequence at t_s and go through the whole interval until the end at t_e. The sub waves of the other spectral algorithms have only a very short length, many times the sub wave disappears within one period length, because of the fixed amplitude and frequency. Therefore the analysis by EMD produces a more natural and appropriate way for market prices. Two important properties can be harvested by the EMD: the average period length of the sub waves and the average amplitude of these sub waves. This information allows to determine the major sub waves of the price sequence. These sub waves are quite consistent and their average period lengths as well as their average amplitudes change only slightly between two consecutive points of time. For real time trading, band-pass filters with the period length of the major sub waves, calculated by the EMD, can be used in order to get consistent and causal trading signals.

V. Conclusion

The empirical mode decomposition (EMD) dismantles the stock market price into a set of intrinsic mode functions (IMF), each of which having an amplitude modulation and a frequency modulation. As a non causal indicator the EMD need to be used with caution if used for generating trading signals, back tests or predicting future prices. The main advantage of the EMD in the context of technical analysis is the calculation of the period length and the amplitudes of the major sub waves, which can not be achieved by the standard spectral algorithms using fixed frequencies and amplitudes. These properties promotes the EMD to a valuable and exceptional tool in the technical analysis.

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